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SC-4305

M. C. A. (Sem. I) (A.T.K.T.) Examination

April / May - 2011

105 - Mathematical Foundation in Computer Science

Time : **3** Hours] [Total Marks : **70**

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.

Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. C. A. (SEM. 1) (A.T.K.T.)**

Name of the Subject :

☛ **105 - MATHEMATICAL FOUNDATION IN COMPUTER SCIENCE**

☛ Subject Code No. : **4 3 0 5** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Attempt all five questions.

(3) Questions numbered 1, 2, 3 carry 16 marks each.

(4) Questions numbered 4 and 5 carry 12 and 10 marks respectively.

1. Answer any **four** of the following:

(i) A can hit a target 3 times in 5 shots, B 2 times in 5 shots, and C 3 times in 4 shots. They fire a volley. What is the probability that (a) two shots hit, (bi) at least two shots hit?

(ii) A company manufactures integrated circuits on silicon chips at three different plants, X, Y and Z. Out of every 1000 chips produced, 400 come from X, 350 come from Y, and 250 come from Z. It has been estimated that of the 400 from X, ten are defective, where as five of those from Y are defective, and only two of those from Z are defective. Determine probability that a defective chip came from plant Y.

(iii) Define rank correlation. Compute rank correlation coefficient between X and Y from the following data:

X :	78	36	98	25	75	82	70	62	65	39
Y :	84	51	91	60	68	62	58	58	53	47

(iv) Find the two regression equations from the following data:

Age of Husband:	28	29	30	31	32	33	34	35	36	37
Age of Wife :	27	27	28	28	29	29	29	30	31	32

Estimate the age of wife, when age of husband is 38 years.

(v) Define binomial distribution. Obtain its mean and variance.

(vi) The probability that an item produced by a company will be defective is 0.01. If 15 such items are produced, find the probability that
(a) Exactly one will be defective,
(b) Exactly two will be defective.

2. Answer any **four** of the following:

(i) If $A = \begin{bmatrix} 3 & -1 & 0 \\ 3 & 2 & -4 \\ 5 & 1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 17 & -1 & 3 \\ -24 & -1 & -16 \\ -7 & 1 & 0 \end{bmatrix}$

Compute (a) AB, (b) A^TB (c) A^TA (d) B^TB

(ii)(a) Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(b) Prove that $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3-1)^2.$

(iii) Define determinant of a matrix $A = (a_{ij})_{n \times n}$.
Find the determinant of the following matrix

$$\begin{bmatrix} 2 & 2 & 5 & -7 \\ 4 & 1 & 5 & 2 \\ 3 & 1 & 7 & -9 \\ 4 & 4 & 2 & 3 \end{bmatrix}$$

(iv) For the matrix A given below show that $A^3 - 2A^2 - 7A - 4I = 0$. Hence obtain A^{-1}.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

(v) Given that $X + Y = A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ 2 & 5 & 8 \end{bmatrix}$
where X is symmetric matrix
and Y is skew-symmetric matrix, then find matrices X and Y.

(vi) Solve the following system of equations, if it exists:

$$\begin{aligned} x + y + z &= 6 \\ 2x + y + 3z &= 11 \\ x - y + z &= 2. \end{aligned}$$

3. Answer any **four** of the following:

(i) Prove that every connected graph has at least one spanning tree.

(ii) Prove that in a simple graph with n vertices, the maximum degree of any vertex is (n-1) and the maximum number of edges is n(n-1)/2.

(iii) Show that a graph G with n vertices, (n-1) edges and no circuit is connected.

(iv) Define a bipartite graph. Draw any two bipartite graphs having at least 8 vertices and check whether they are isomorphic or not.

(v) Explain when two graphs are said to be isomorphic. State the necessary conditions for two graphs to be isomorphic. Give an example involving two graphs having at least 5 vertices to show that these conditions are not sufficient conditions.

(vi) Define radius and diameter of a tree. Prove that every tree has either one or two centres.

(vi) Define height of a binary tree. Show that minimum possible height h of an n-vertex tree is given by the relation

$$\log_2(n+1) \leq (h+1).$$

4. Answer any **three** of the following:

(i) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then compute $(AB)^{-1}$.

(ii) Show that the necessary and sufficient condition for a graph G to be a tree is that there is one and only one path between every pair of vertices in G.

(iii) Describe utilities problem and discuss its relevance in graph theory.

(iv) 25 pairs of value of variates X and Y led to the following results:

$n=25$, $\sum X = 127$, $\sum Y = 100$, $\sum X^2 = 760$, $\sum Y^2 = 449$, $\sum XY = 500$,

A subsequent scrutiny showed that two pairs of values were copied down as:

X	Y	instead of	X	Y
8	14		8	12
8	6		6	8

Obtain the correct values of the correlation coefficient.

(v) If the distribution of incomes of a group of persons be assumed to be normal with mean 500 and the standard deviation Rs. 50. Estimate the proportion of individuals with income (a) below Rs. 550, and (b) between Rs. 550 and Rs. 650.

5. Answer any **two** of the following:

(i) $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Then verify the following:

(a) $A(B-C) = AB - AC$, (b) $(A+B)x(A-B) = A^2 - B^2$.

(ii) Define height of a binary tree. Show that minimum possible height h of an n-vertex tree is given by the relation

$$\log_2(n+1) \leq (h+1).$$

(iii) Show that for poisson distribution mean and variance are same.